

## Basic Formulas

$$1. \int ax^n dx = \frac{a}{n+1} x^{n+1}$$

$$2. \int \frac{dx}{ax+b} = \frac{1}{a} \ln |ax+b|$$

$$3. \int ab^x dx = \frac{1}{\ln b} ab^x$$

$$4. \int \sin ax dx = -\frac{\cos ax}{a}$$

$$5. \int \cos ax dx = \frac{\sin ax}{a}$$

$$6. \int \tan ax dx = \frac{\ln |\cos ax|}{a}$$

$$7. \int u dv = uv - \int v du$$

$$8. \sin(x) = \cos(x - \frac{\pi}{2})$$

$$9. \sin^2(x) = 1 - \cos^2(x)$$

$$10. \sin(2x) = 2 \sin(x) \cos(x)$$

$$11. \cos(2x) = 2 \cos^2(x) - 1$$

$$12. \tan(x) = \frac{1 - \cos(2x)}{\sin(2x)}$$

$$13. a \sin(x) + b \cos(x) =$$

$$\sqrt{a^2 + b^2} \sin(x + \tan^{-1} \frac{b}{a})$$

## Rational Functions

$$14. \int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \ln |ax+b|$$

$$15. \int \frac{x^2}{ax+b} dx = \frac{x^2}{2a} - \frac{bx}{a^2} - \frac{3b^2}{a^3} +$$

$$\frac{b^2}{a^3} \ln |ax+b|$$

$$16. \int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

$$17. \int \frac{x}{(ax+b)^2} dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \ln |ax+b|$$

$$18. \int \frac{x^2}{(ax+b)^2} dx = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} -$$

$$\frac{2b}{a^3} \ln |ax+b|$$

$$19. \int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln \left| \frac{x}{ax+b} \right|$$

$$20. \int \frac{dx}{x^2(ax+b)} = \frac{a}{b^2} \ln \left| \frac{ax+b}{x} \right| - \frac{1}{bx}$$

$$21. \int \frac{dx}{x^2(ax+b)^2} = \frac{2a}{b^3} \ln \left| \frac{ax+b}{x} \right| - \frac{2ax+b}{b^2x(ax+b)}$$

$$22. \int \frac{dx}{x(x^2+a^2)} = \frac{1}{2a^2} \ln \left( \frac{x^2}{x^2+a^2} \right)$$

$$23. \int \frac{dx}{x^2(x^2+a^2)} = \frac{-1}{a^2x} - \frac{1}{a^3} \tan^{-1} \left( \frac{x}{a} \right)$$

$$24. \int \frac{x^2}{(x^2+a^2)^2} dx = \frac{1}{2a} \tan^{-1} \left( \frac{x}{a} \right) - \frac{x}{2(x^2+a^2)}$$

$$25. \int \frac{dx}{x(x^2-a^2)} = \frac{1}{2a^2} \ln \left| \frac{x^2-a^2}{x^2} \right|$$

$$26. \int \frac{dx}{x^2(x^2-a^2)} = \frac{1}{a^2x} + \frac{1}{2a^3} \ln \left| \frac{x-a}{x+a} \right|$$

$$27. \int \frac{x^2}{(x^2-a^2)^2} dx = \frac{x}{2(a^2-x^2)} - \frac{1}{4a} \ln \left| \frac{x+a}{x-a} \right|$$

$$28. \int \frac{dx}{x^2+ax+b} =$$

$$\begin{cases} \frac{2}{\sqrt{4b-a^2}} \tan^{-1} \left( \frac{2x+a}{\sqrt{4b-a^2}} \right) & \text{if } 4b > a^2 \\ \frac{-1}{x+a/2} & \text{if } 4b = a^2 \\ \frac{1}{\sqrt{a^2-4b}} \ln \left| \frac{2x+a-\sqrt{a^2-4b}}{2x+a+\sqrt{a^2-4b}} \right| & \text{if } 4b < a^2 \end{cases}$$

$$29. \int \frac{x}{x^2+ax+b} dx = \frac{1}{2} \ln |x^2+ax+b| +$$

$$\begin{cases} \frac{a}{\sqrt{4b-a^2}} \tan^{-1} \left( \frac{2x+a}{\sqrt{4b-a^2}} \right) & \text{if } 4b > a^2 \\ \frac{a}{2x+a} & \text{if } 4b = a^2 \\ \frac{-a}{2\sqrt{a^2-4b}} \ln \left| \frac{2x+a-\sqrt{a^2-4b}}{2x+a+\sqrt{a^2-4b}} \right| & \text{if } 4b < a^2 \end{cases}$$

$$30. \int \frac{x^2}{x^2+ax+b} dx = x - \frac{a}{2} \ln |x^2+ax+b| +$$

$$\begin{cases} \frac{a^2-2b}{\sqrt{4b-a^2}} \tan^{-1} \left( \frac{2x+a}{\sqrt{4b-a^2}} \right) & \text{if } 4b > a^2 \\ \frac{-a^2}{4x+2a} & \text{if } 4b = a^2 \\ \frac{a^2-2b}{2\sqrt{a^2-4b}} \ln \left| \frac{2x+a-\sqrt{a^2-4b}}{2x+a+\sqrt{a^2-4b}} \right| & \text{if } 4b < a^2 \end{cases}$$

$$31. \int \frac{dx}{x^3+a^3} = \frac{1}{6a^2} \ln \left| \frac{x^2+2ax+a^2}{x^2-ax+a^2} \right| + \frac{1}{\sqrt{3}a^2} \tan^{-1} \left( \frac{2x-a}{\sqrt{3}a} \right)$$

$$32. \int \frac{x}{x^3+a^3} dx = \frac{1}{6a} \ln \left| \frac{x^2-ax+a^2}{x^2+2ax+a^2} \right| + \frac{1}{\sqrt{3}a} \tan^{-1} \left( \frac{2x-a}{\sqrt{3}a} \right)$$

$$33. \int \frac{dx}{x^4+a^4} = \frac{1}{4\sqrt{2}a^3} \ln \left| \frac{x^2+\sqrt{2}ax+a^2}{x^2-\sqrt{2}ax+a^2} \right| + \frac{1}{2\sqrt{2}a^3} \tan^{-1} \left( \frac{\sqrt{2}a}{a^2-x^2} \right)$$

$$34. \int \frac{x}{x^4+a^4} dx = \frac{-1}{2a^2} \tan^{-1} \left( \frac{a^2}{x^2} \right)$$

$$35. \int \frac{x^2}{x^4+a^4} dx = \frac{1}{4\sqrt{2}a} \ln \left| \frac{x^2-\sqrt{2}ax+a^2}{x^2+\sqrt{2}ax+a^2} \right| + \frac{1}{2\sqrt{2}a} \tan^{-1} \left( \frac{\sqrt{2}a}{a^2-x^2} \right)$$

### Square Roots

$$36. \int \sqrt{ax+b} dx = \frac{2}{3a}(ax+b)^{3/2}$$

$$37. \int \frac{dx}{\sqrt{ax+b}} = \frac{2}{a}\sqrt{ax+b}$$

$$38. \int x\sqrt{ax+b} dx = \frac{6ax-4b}{15a^2}(ax+b)^{3/2}$$

$$39. \int \frac{x}{\sqrt{ax+b}} dx = \frac{2ax-4b}{3a^2}\sqrt{ax+b}$$

$$40. \int \frac{\sqrt{ax+b}}{x} dx = 2\sqrt{ax+b} + \begin{cases} \sqrt{b} \ln \left| \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}} \right| & \text{if } b > 0 \\ -2\sqrt{-b} \tan^{-1} \left( \sqrt{\frac{ax+b}{-b}} \right) & \text{if } b < 0 \end{cases}$$

$$41. \int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}} \right| & \text{if } b > 0 \\ \frac{2}{\sqrt{-b}} \tan^{-1} \left( \sqrt{\frac{ax+b}{-b}} \right) & \text{if } b < 0 \end{cases}$$

$$42. \int \sqrt{\frac{ax+b}{rx+s}} dx = \frac{2(as-br)}{r^2} \int \frac{u^2}{(u^2-a/c)^2} du$$

where  $u = \sqrt{\frac{ax+b}{rx+s}}$ , see eqs. 24 & 27

$$43. \int \frac{dx}{a\sqrt{x+b}} = \frac{2\sqrt{x}}{a} - \frac{2b}{a^2} \ln |a\sqrt{x} + b|$$

$$44. \int \frac{\sqrt{x}}{a\sqrt{x+b}} dx = \frac{x}{a} - \frac{2b\sqrt{x}}{a^2} + \frac{2b^2}{a^3} \ln |a\sqrt{x} + b|$$

$$45. \int \frac{x}{a\sqrt{x+b}} dx = \frac{2}{3a}x^{3/2} - \frac{bx}{a^2} + \frac{2b^2}{a^3}\sqrt{x} -$$

$$\frac{2b^3}{a^4} \ln |a\sqrt{x} + b|$$

$$46. \int \sqrt{x+a\sqrt{x}+b} dx =$$

$$\left( \frac{2}{3}x - \frac{a}{6}\sqrt{x} + \frac{2b}{3} - \frac{a^2}{4} \right) (x+a\sqrt{x}+b)^{3/2} -$$

$$\left( ab - \frac{a^3}{4} \right) \ln \left| \sqrt{x} + \frac{a}{2} + \sqrt{x+a\sqrt{x}+b} \right|$$

$$47. \int \sqrt{x^2+a^2} dx = \frac{1}{2}x\sqrt{x^2+a^2} +$$

$$\frac{a^2}{2} \ln |x + \sqrt{x^2+a^2}|$$

$$48. \int \sqrt{x^2-a^2} dx = \frac{1}{2}x\sqrt{x^2-a^2} -$$

$$\frac{a^2}{2} \ln |x + \sqrt{x^2-a^2}|$$

$$49. \int \sqrt{a^2-x^2} dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \left| \frac{x}{a} \right|$$

$$50. \int x^2\sqrt{x^2+a^2} dx =$$

$$\frac{2x^3+a^2x}{8}\sqrt{x^2+a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2+a^2}|$$

$$51. \int x^2\sqrt{x^2-a^2} dx =$$

$$\frac{2x^3-a^2x}{8}\sqrt{x^2-a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2-a^2}|$$

$$52. \int x^2 \sqrt{a^2 - x^2} dx = \frac{2x^3 - a^2 x}{8} \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \left( \frac{x}{a} \right)$$

$$53. \int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|$$

$$54. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left( \frac{x}{a} \right)$$

$$55. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$56. \int \frac{\sqrt{x^2 + a^2}}{x^2} dx = -\frac{\sqrt{x^2 + a^2}}{x} + \ln \left| x + \sqrt{x^2 + a^2} \right|$$

$$57. \int \frac{\sqrt{x^2 - a^2}}{x^2} dx = -\frac{\sqrt{x^2 - a^2}}{x} + \ln \left| x + \sqrt{x^2 - a^2} \right|$$

$$58. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1} \left( \frac{x}{a} \right)$$

$$59. \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left| x + \sqrt{x^2 + a^2} \right|$$

$$60. \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right|$$

$$61. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left( \frac{x}{a} \right)$$

$$62. \int \frac{x^2}{\sqrt{x^2 + a^2}} dx = \frac{1}{2} x \sqrt{x^2 + a^2} - \frac{a^2}{2} \ln \left| x + \sqrt{x^2 + a^2} \right|$$

$$63. \int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{1}{2} x \sqrt{x^2 - a^2} + \frac{a^2}{2} \ln \left| x + \sqrt{x^2 - a^2} \right|$$

$$64. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right)$$

$$65. \int \frac{dx}{x \sqrt{x^2 + a^2}} = \frac{-1}{a} \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|$$

$$66. \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right|$$

$$67. \int \frac{dx}{x \sqrt{a^2 - x^2}} = \frac{-1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$68. \int \frac{dx}{x^2 \sqrt{x^2 + a^2}} = \frac{-\sqrt{x^2 + a^2}}{a^2 x}$$

$$69. \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x}$$

$$70. \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = \frac{-\sqrt{a^2 - x^2}}{a^2 x}$$

$$71. \int \frac{dx}{x + \sqrt{x^2 + a^2}} = \frac{x}{2a^2} \sqrt{x^2 + a^2} - \frac{x^2}{2a^2} + \frac{1}{2} \ln(x + \sqrt{x^2 + a^2})$$

$$72. \int \frac{dx}{x + \sqrt{x^2 - a^2}} = \frac{-x}{2a^2} \sqrt{x^2 - a^2} + \frac{x^2}{2a^2} + \frac{1}{2} \ln(x + \sqrt{x^2 - a^2})$$

$$73. \int \frac{dx}{x + \sqrt{a^2 - x^2}} = \ln(x + \sqrt{a^2 - x^2}) - \frac{1}{4} \ln |2x^2 - a^2| + \frac{1}{2} \tan^{-1} \left( \frac{x}{\sqrt{a^2 - x^2}} \right)$$

$$74. \int \frac{dx}{a + \sqrt{a^2 - x^2}} = \frac{\sqrt{a^2 - x^2}}{x} + \tan^{-1} \left( \frac{x}{\sqrt{a^2 - x^2}} \right) - \frac{a}{x}$$

### Natural Logarithms

$$75. \int x^n \ln x dx = \frac{x^{n+1} \ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2}, \quad n \neq -1$$

$$76. \int \frac{\ln x}{x} dx = \frac{1}{2} (\ln x)^2$$

$$77. \int \frac{(\ln x)^m}{x} dx = \frac{(\ln x)^{m+1}}{m+1}, \quad m \neq -1$$

$$78. \int \frac{dx}{x \ln x} = \ln |\ln x|$$

$$79. \int \frac{\ln x}{(x+a)^2} dx = \frac{1}{a} \left( \frac{x}{x+a} \ln x - \ln |x+a| \right)$$

$$80. \int (\ln x)^2 dx = x [(\ln x)^2 - 2 \ln x + 2]$$

$$81. \int \ln(a\sqrt{x} + b) dx =$$

$$\left(x - \frac{b^2}{a^2}\right) \ln(a\sqrt{x} + b) - \frac{x}{2} + \frac{b\sqrt{x}}{a}$$

$$82. \int \ln(x^2 + ax + b) dx = -2x - a +$$

$$\left(x + \frac{a}{2}\right) \ln(x^2 + ax + b) +$$

$$\begin{cases} \sqrt{4b - a^2} \tan^{-1} \left( \frac{2x}{\sqrt{4b - a^2}} \right) & \text{if } 4b > a^2 \\ 0 & \text{if } 4b = a^2 \\ \frac{1}{2} \sqrt{a^2 - 4b} \ln \left| \frac{2x + a + \sqrt{a^2 - 4b}}{2x + a - \sqrt{a^2 - 4b}} \right| & \text{if } 4b < a^2 \end{cases}$$

$$83. \int \ln(x + \sqrt{x^2 + a^2}) dx = -\sqrt{x^2 + a^2} +$$

$$x \ln(x + \sqrt{x^2 + a^2})$$

$$84. \int \ln(x + \sqrt{x^2 - a^2}) dx = -\sqrt{x^2 - a^2} +$$

$$x \ln(x + \sqrt{x^2 - a^2})$$

$$85. \int \ln(x - \sqrt{x^2 - a^2}) dx = 2x \ln a +$$

$$\sqrt{x^2 - a^2} - x \ln(x + \sqrt{x^2 - a^2})$$

$$86. \int \ln(x + \sqrt{a^2 - x^2}) dx =$$

$$x \ln(x + \sqrt{a^2 - x^2}) - x -$$

$$\frac{a}{2} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$87. \int \ln(a + \sqrt{a^2 - x^2}) dx = x +$$

$$x \ln(a + \sqrt{a^2 - x^2}) - a \sin^{-1} \left( \frac{x}{a} \right)$$

$$88. \int \ln(x^3 + a^3) dx = x \ln(x^3 + a^3) - x +$$

$$\frac{a}{2} \ln \left| \frac{x^2 + 2ax + a^2}{x^2 - ax + a^2} \right| + \sqrt{3}a \tan^{-1} \left( \frac{2x}{\sqrt{3}a} \right)$$

## Exponential Functions

$$89. \int x e^{ax} dx = \frac{x e^{ax}}{a} - \frac{e^{ax}}{a^2}$$

$$90. \int x^2 e^{ax} dx = \frac{x^2 e^{ax}}{a} - \frac{2x e^{ax}}{a^2} + \frac{2e^{ax}}{a^3}$$

$$91. \int e^{a\sqrt{x}} dx = \frac{2}{a} \sqrt{x} e^{a\sqrt{x}} - \frac{2}{a^2} e^{a\sqrt{x}}$$

$$92. \int \frac{dx}{b + e^{ax}} = \frac{x}{b} - \frac{1}{ab} \ln |b + e^{ax}|$$

$$93. \int \frac{e^{ax}}{b + e^{ax}} dx = \frac{1}{a} \ln |b + e^{ax}|$$

$$94. \int \sqrt{e^{ax} + b} dx = \frac{2}{a} \sqrt{e^{ax} + b} +$$

$$\begin{cases} \frac{\sqrt{b}}{a} \ln \left| \frac{\sqrt{e^{ax} + b} - \sqrt{b}}{\sqrt{e^{ax} + b} + \sqrt{b}} \right| & \text{if } b > 0 \\ -\frac{2\sqrt{-b}}{a} \tan^{-1} \left( \frac{\sqrt{e^{ax} + b}}{\sqrt{-b}} \right) & \text{if } b < 0 \end{cases}$$

$$95. \int \sqrt{b - e^{ax}} dx = \frac{2}{a} \sqrt{b - e^{ax}} +$$

$$\frac{\sqrt{b}}{a} \ln \left| \frac{\sqrt{e^{ax} + b} - \sqrt{b}}{\sqrt{e^{ax} + b} + \sqrt{b}} \right|$$

$$96. \int \frac{dx}{\sqrt{b - e^{ax}}} = \frac{\sqrt{b}}{a} \ln \left| \frac{\sqrt{b - e^{ax}} - \sqrt{b}}{\sqrt{b - e^{ax}} + \sqrt{b}} \right|$$

$$97. \int \frac{dx}{\sqrt{e^{ax} + b}} =$$

$$\begin{cases} \frac{\sqrt{b}}{a} \ln \left| \frac{\sqrt{e^{ax} + b} - \sqrt{b}}{\sqrt{e^{ax} + b} + \sqrt{b}} \right| & \text{if } b > 0 \\ \frac{2\sqrt{-b}}{a} \tan^{-1} \left( \frac{\sqrt{e^{ax} + b}}{\sqrt{-b}} \right) & \text{if } b < 0 \end{cases}$$

$$98. \int e^{ax} \sin(bx) dx =$$

$$\frac{e^{ax}}{a^2 + b^2} [a \sin(bx) - b \cos(bx)]$$

$$99. \int e^{ax} \cos(bx) dx =$$

$$\frac{e^{ax}}{a^2 + b^2} [b \sin(bx) + a \cos(bx)]$$

$$100. \int x e^{ax} \sin(bx) dx = \frac{e^{ax}}{a^2+b^2} [ax \sin(bx) - bx \cos(bx) - \frac{a^2-b^2}{a^2+b^2} \sin(bx) + \frac{2ab}{a^2+b^2} \cos(bx)]$$

$$101. \int x e^{ax} \cos(bx) dx = \frac{e^{ax}}{a^2+b^2} [bx \sin(bx) + ax \cos(bx) - \frac{a^2-b^2}{a^2+b^2} \cos(bx) - \frac{2ab}{a^2+b^2} \sin(bx)]$$

### Trigonometric Functions

$$102. \int \sin^2(x) dx = \frac{x}{2} - \frac{\sin(2x)}{4}$$

$$103. \int \sin^3(x) dx = -\sin^2(x) \cos(x) - \frac{2}{3} \cos^3(x)$$

$$104. \int \cos^2(x) dx = \frac{x}{2} + \frac{\sin(2x)}{4}$$

$$105. \int \cos^3(x) dx = \sin(x) \cos^2(x) + \frac{2}{3} \sin^3(x)$$

$$106. \int \sin^2(x) \cos^2(x) dx = \frac{x}{16} - \frac{\sin(4x)}{32}$$

$$107. \int x \sin(x) dx = \sin(x) - x \cos(x)$$

$$108. \int x^2 \sin(x) dx = 2 \cos(x) +$$

$$2x \sin(x) - x^2 \cos(x)$$

$$109. \int x \sin^2(x) dx = \frac{x^2}{4} - \frac{x \sin(2x)}{4} - \frac{\cos(2x)}{8}$$

$$110. \int x^2 \sin^2(x) dx = \frac{x^3}{6} - \frac{x^2 \sin(2x)}{4} -$$

$$\frac{x \cos(2x)}{4} + \frac{\sin(2x)}{8}$$

$$111. \int x \cos(x) dx = \cos(x) + x \sin(x)$$

$$112. \int x^2 \cos(x) dx = -2 \sin(x) +$$

$$2x \cos(x) + x^2 \sin(x)$$

$$113. \int x \cos^2(x) dx = \frac{x^2}{4} + \frac{x \sin(2x)}{4} + \frac{\cos(2x)}{8}$$

$$114. \int x^2 \cos^2(x) dx = \frac{x^3}{6} + \frac{x^2 \sin(2x)}{4} +$$

$$\frac{x \cos(2x)}{4} - \frac{\sin(2x)}{8}$$

$$115. \int \frac{dx}{\sin(x)} = \ln \left| \frac{1-\cos(x)}{\sin(x)} \right|$$

$$116. \int \frac{dx}{\cos(x)} = \ln \left| \frac{1+\sin(x)}{\cos(x)} \right|$$

$$117. \int \frac{dx}{\sin^2(x)} = -\cot(x)$$

$$118. \int \frac{dx}{\cos^2(x)} = \tan(x)$$

$$119. \int \frac{dx}{\sin^3(x)} = \frac{-\cos^2(x)}{2 \sin(x)} + \frac{1}{2} \ln \left| \frac{1-\cos(x)}{\sin(x)} \right|$$

$$120. \int \frac{dx}{\cos^3(x)} = \frac{\sin^2(x)}{2 \cos(x)} + \frac{1}{2} \ln \left| \frac{1+\sin(x)}{\cos(x)} \right|$$

$$121. \int \frac{dx}{\sin^2(x) \cos(x)} = \ln \left| \frac{1+\sin(x)}{\cos(x)} \right| - \frac{1}{\sin(x)}$$

$$122. \int \frac{dx}{\sin(x) \cos^2(x)} = \ln \left| \frac{1-\cos(x)}{\sin(x)} \right| + \frac{1}{\cos(x)}$$

$$123. \int \sin(x) \sin(x+a) dx = \frac{1}{2} x \cos(a) - \frac{1}{4} \sin(2x+a)$$

$$124. \int \sin(ax) \sin(bx) dx = \frac{\sin((a-b)x)}{2(a-b)} - \frac{\sin((a+b)x)}{2(a+b)}$$

$$125. \int \cos(ax) \cos(bx) dx = \frac{\sin((a-b)x)}{2(a-b)} + \frac{\sin((a+b)x)}{2(a+b)}$$

$$126. \int \sin(ax) \cos(bx) dx = \frac{-\cos((a-b)x)}{2(a-b)} - \frac{\cos((a+b)x)}{2(a+b)}$$

$$127. \int \frac{dx}{1 \pm \sin(x)} = \mp \tan \left( \frac{\pi}{4} \mp \frac{x}{2} \right)$$

$$128. \int \frac{dx}{1 \pm \cos(x)} = \mp \tan\left(\frac{\pi}{4} \mp \left(\frac{2x+\pi}{4}\right)\right)$$

$$129. \int \frac{x}{1 \pm \sin(x)} dx = x \tan\left(\frac{\pi}{4} \mp \frac{x}{2}\right) \mp 2 \ln \left| \sin\left(\frac{\pi}{4} \pm \frac{x}{2}\right) \right|$$

$$131. \int \frac{dx}{a + \sin(x)} = \begin{cases} \frac{2}{\sqrt{a^2-1}} \tan^{-1} \left[ \frac{\sqrt{a^2-1}}{|a-1|} \tan\left(\frac{2x+\pi}{4}\right) \right] & \text{if } |a| > 1 \\ \frac{1}{\sqrt{1-a^2}} \ln \left| \frac{a \tan(x/2) + 1 - \sqrt{1-a^2}}{a \tan(x/2) + 1 + \sqrt{1-a^2}} \right| & \text{if } |a| < 1 \end{cases}$$

$$132. \int \frac{dx}{a + \cos(x)} = \begin{cases} \frac{2}{\sqrt{a^2-1}} \tan^{-1} \left[ \frac{|a-1|}{\sqrt{a^2-1}} \tan\left(\frac{x}{2}\right) \right] & \text{if } |a| > 1 \\ \frac{1}{\sqrt{1-a^2}} \ln \left| \frac{\tan(x/2) + \frac{\sqrt{1-a^2}}{1-a}}{\tan(x/2) - \frac{\sqrt{1-a^2}}{1-a}} \right| & \text{if } |a| < 1 \end{cases}$$

$$133. \int \frac{dx}{a \sin(x) + \cos(x)} = \frac{1}{\sqrt{a^2+1}} \ln \left| \tan\left(\frac{x + \tan^{-1} \frac{1}{a}}{2}\right) \right|$$

$$134. \int \frac{\cos(x)}{a \sin(x) + \cos(x)} dx = \frac{x}{a^2+1} + \frac{a}{2(a^2+1)} \ln \left| a + \frac{a^2+1}{2} \sin(2x) \right| + \frac{a}{a^2+1} \ln \left| \frac{a \tan(x)+1}{a \tan(x)+a^2} \right|$$

$$135. \int \tan^2(x) dx = \tan(x) - x$$

$$136. \int \tan^3(x) dx = \frac{1}{2} \tan^2(x) + \ln |\cos(x)|$$

$$137. \int \cot^2(x) dx = -\cot(x) - x$$

$$138. \int \cot^3(x) dx = -\frac{1}{2} \cot^2(x) - \ln |\sin(x)|$$

$$139. \int \sin^{-1}(x) dx = x \sin^{-1}(x) + \sqrt{1-x^2}$$

$$140. \int (\sin^{-1}(x))^2 dx = x(\sin^{-1}(x))^2 -$$

$$2x + 2 \sin^{-1}(x) \sqrt{1-x^2}$$

$$141. \int \sin^{-1}\left(\frac{1}{x}\right) dx = x \sin^{-1}\left(\frac{1}{x}\right) +$$

$$\ln(x + \sqrt{x^2-1})$$

$$142. \int \cos^{-1}(x) dx = x \cos^{-1}(x) - \sqrt{1-x^2}$$

$$130. \int \frac{x}{1 \pm \cos(x)} dx = (x + \frac{\pi}{2}) \tan\left(\frac{\pi}{4} \mp \left(\frac{2x+\pi}{4}\right)\right) \mp$$

$$2 \ln \left| \sin\left(\frac{\pi}{4} \pm \left(\frac{2x+\pi}{4}\right)\right) \right|$$

$$143. \int (\cos^{-1}(x))^2 dx = x(\cos^{-1}(x))^2 -$$

$$2x - 2 \cos^{-1}(x) \sqrt{1-x^2}$$

$$144. \int \cos^{-1}\left(\frac{1}{x}\right) dx = x \cos^{-1}\left(\frac{1}{x}\right) -$$

$$\ln(x + \sqrt{x^2-1})$$

$$145. \int (\sin^{-1}(x))(\cos^{-1}(x)) dx =$$

$$x \sin^{-1}(x) \cos^{-1}(x) + 2x +$$

$$\sqrt{1-x^2}(\cos^{-1}(x) - \sin^{-1}(x))$$

$$146. \int \sin(a \sin^{-1}(x)) dx =$$

$$\frac{a\sqrt{1-x^2} \cos(a \sin^{-1}(x)) + x \sin(a \sin^{-1}(x))}{1-a^2}$$

$$147. \int \cos(a \cos^{-1}(x)) dx =$$

$$\frac{a\sqrt{1-x^2} \sin(a \cos^{-1}(x)) + x \cos(a \cos^{-1}(x))}{1-a^2}$$

$$148. \int \tan^{-1}(x) dx = x \tan^{-1}(x) -$$

$$\frac{1}{2} \ln(x^2 + 1)$$

$$153. \int \tan^{-1}\left(\frac{ax+b}{x+c}\right) dx = \left(x + \frac{ab+c}{a^2+1}\right) \tan^{-1}\left(\frac{ax+b}{x+c}\right) + \frac{b-ac}{2a^2+2} \ln((ax+b)^2 + (x+c)^2)$$

$$149. \int \tan(a + \tan^{-1}(x)) dx =$$

$$x \cot(a) - \frac{1}{\sin^2(a)} \ln(x \sin(a) - \cos(a))$$

$$150. \int x \sin^{-1}(x) dx = \frac{(2x^2-1) \sin^{-1} x + x\sqrt{1-x^2}}{4}$$

$$151. \int x \cos^{-1}(x) dx = \frac{(2x^2-1) \cos^{-1} x - x\sqrt{1-x^2}}{4}$$

$$152. \int x \tan^{-1}(x) dx = \frac{(x^2+1) \tan^{-1}(x) - x}{2}$$

### Hyperbolic Functions

$$154. \sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$155. \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$156. \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$157. \cosh^2(x) - \sinh^2(x) = 1$$

$$158. \sinh^{-1} = \ln(x + \sqrt{x^2 + 1})$$

$$159. \cosh^{-1} = \ln(x + \sqrt{x^2 - 1})$$

$$160. \tanh^{-1} = \ln\left(\frac{1+y}{1-y}\right)$$

$$161. \int \sinh(x) dx = \cosh(x)$$

$$162. \int \cosh(x) dx = \sinh(x)$$

$$163. \int \tanh(x) dx = \ln(e^x + e^{-x})$$

$$164. \int \frac{dx}{\sinh(x)} = \ln\left|\frac{e^x-1}{e^x+1}\right|$$

$$165. \int \frac{dx}{\cosh(x)} = 2 \tan^{-1}(e^x)$$

$$166. \int \frac{dx}{\sinh^2(x)} = -\coth(x) = \frac{-1}{\tanh(x)}$$

$$167. \int \frac{dx}{\cosh^2(x)} = \tanh(x)$$

$$168. \int \tanh^2(x) dx = x - \tanh(x)$$

$$169. \int x \sinh(x) dx = x \cosh(x) - \sinh(x)$$

$$170. \int x \cosh(x) dx = x \sinh(x) - \cosh(x)$$

$$171. \int \sinh^{-1}(x) dx = x \sinh^{-1}(x) -$$

$$\sqrt{x^2 + 1}$$

$$172. \int \cosh^{-1}(x) dx = x \cosh^{-1}(x) -$$

$$\sqrt{x^2 - 1}$$

$$173. \int \tanh^{-1}(x) dx = \frac{1}{2} \ln(1 - x^2) +$$

$$x \tanh^{-1}(x)$$

$$174. \int x \sinh^{-1}(x) dx =$$

$$\frac{2x^2+1}{4} \sinh^{-1}(x) - \frac{1}{4} \sqrt{x^2 + 1}$$

$$175. \int x \cosh^{-1}(x) dx =$$

$$\frac{2x^2+1}{4} \cosh^{-1}(x) - \frac{1}{4} \sqrt{x^2 - 1}$$

$$176. \int x \tanh^{-1}(x) dx = \frac{1}{2}(x^2 - 1) \tanh^{-1}(x) + \frac{x}{2}$$

$$177. \int (\sinh^{-1}(x))^2 dx = 2x + x(\sinh^{-1}(x))^2 - 2 \sinh^{-1}(x)\sqrt{x^2 + 1}$$

$$178. \int (\cosh^{-1}(x))^2 dx = 2x + x(\cosh^{-1}(x))^2 - 2 \cosh^{-1}(x)\sqrt{x^2 - 1}$$

$$179. \int \sinh^{-1}\left(\frac{1}{x}\right) dx = x \sinh^{-1}\left(\frac{1}{x}\right) + \sinh^{-1}(x)$$

$$180. \int \cosh^{-1}\left(\frac{1}{x}\right) dx = x \cosh^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$